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OCEAN ACOUSTIC PROPAGATION PREDICTIONS FROM AVERAGED SOUND SPEEDS COMPARED TO INDIVIDUAL SOUND SPEEDS

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Abstract: Wave propagation through volume randomness models much ocean acoustic behavior. Random sound speed introduces non-linearity in the differential equations for the ensemble averages. This leads to results which can be significantly different than if an averaged volume parameters, such as sound speed, are used in a deterministic equation. We examine some theoretical aspects of randomness in the normal mode and parabolic equations. As an example, transmission losses in several specific examples are computed where differences result.

INTRODUCTION

The study of waves through random media has been active for about four decades and has several comprehensive reviews, including Ishimaru (1978) and Flatté (1983) (which emphasizes ocean acoustics). One wishes to know about the average solution to a wave equation with uncertain sound speed. Frisch (1968) points out that this type of problem is nonlinear in its dependence on the coefficients. As an introductory example, consider the one dimensional wave equation with constant sound speed $u_{tt} = c^2 u_{xx}$ with initial values $u(0,x) = f(x)$, $u_t(0,x) = 0$. This problem has solutions that can be expressed $u(t,x) = \frac{1}{2}[f(x-ct) + f(x+ct)]$. A probability distribution of sound speeds give averages of the form $\langle f \rangle = \int f\, dP(c) = \int f\phi(c)dc$. The average solution convolves initial values with a time dependent probability distribution:

$$\langle u\rangle(t,x) = \int f(y)\,\psi_t(y)dy, \quad \psi_t(y) = \frac{1}{2t}\left[\phi\left(\frac{x-y}{t}\right) + \phi\left(\frac{y-x}{t}\right)\right]. \tag{1}$$

For example, the normal distribution $\phi(c) = (\sigma\sqrt{2\pi})^{-1}\exp[-(c-c_0)^2/2\sigma^2]$ and the initial condition, $f(x) = e^{ikx}$, give an average solution $u(t,x) = e^{ikx}\cos kc_0 t \exp(-[k\sigma t]^2/2)$. This shows attenuation of the mean field. The average solution contains information about the uncertainty as decay with time; it behaves differently from the simple oscillatory solution for the averaged sound speed. One can also show saturation phenomenon, *i.e.*, variations comparable to amplitude as t becomes large. In the same case, but given boundary values at $x=0$ and radiation conditions for $x \to \infty$ instead, one can show phase speed of the average solution is less than the average phase speed. Note, however, that the solution is linear in the initial conditions, which means the average of initial conditions gives the solution equal to the average of solutions with those initial conditions.

EXACT SOLUTIONS TO RANGE INDEPENDENT CASE

The source of the non-linearity in this case is the inversion of the differential operator to find the solution. To see this, let us consider the case of a horizontally stratified ocean model with flat pressure release top surface and bottom at $-h$ which models the trapped modes of the Pekeris two fluid model (*cf.* Kinsler *et al.* (1982)). When this is driven by a continuous wave, point source this reduces to a two-dimensional Helmholtz equation, which in axi-symmetric, range independent form is

$$[r^{-1}\partial_r r\partial_r + \partial_z^2 + k^2 n^2(z)] = (2\pi r)^{-1}\delta(z - z_0)\delta(r). \tag{2}$$

Boundary conditions on the top are $u(z=0)=0$ and the bottom $k\cos\theta\, u(t,-h) - \sin\theta\, u_z(t,-h) = 0$. This leads to a completely discrete spectrum. Continuous spectrum can be treated completely analogously, with integration over the spectrum rather than summation. This assumption precludes the more complicated case where a fixed part of the spectrum is discrete in some parts of the probability space and continuous in others. To isolate the non-linear step, use the Hankel transform method in Ahluwalia and Keller (1977). Define the Hankel transform and its inverse by

$$\mathrm{H}f(s) = 2\pi\int_0^\infty J_0(rs)\, r\, f(r)dr, \quad \mathrm{H}^{-1}f(r) = \tfrac{1}{2\pi}\int_0^\infty J_0(rs)\, s\, f(s)ds. \tag{3}$$

In the Hankel transformed equation substitute $s = ka$ to obtain

$$[\partial_z^2 + k^2(n^2 + a^2)]f = \delta(z - z_0). \tag{4}$$

The solutions may now be expressed as inverse Hankel transforms of the the Green's function. First define two fundamental solutions: $f_1(a,z)$ which matches bottom boundary condition and $f_2(a,z)$ which matches top boundary condition. The Green's function solution is

$$G(z, z_0) = W^{-1} f_1(z_<) f_2(z_>), \tag{5}$$

where $z_< = \min(z, z_0)$, $z_> = \max(z, z_0)$ and the Wronskian is $W = W(ka) = f_{1,z} f_2 - f_1 f_{2,z}$. Thus the solution to wave equation is the inverse Hankel transform of Green's function

$$u(r,z) = \mathrm{H}^{-1}G. \tag{6}$$

(6) uses a single z-dependent sound speed profile. The average of solutions for many profiles is

$$\langle u\rangle = E(\mathrm{H}^{-1}G) = \mathrm{H}^{-1}E(G) = \mathrm{H}^{-1}\left\langle W^{-1} f_1 f_2\right\rangle. \tag{7}$$

$E(G)$ is an averaged Green's function, and, obviously, one cannot simply pull the averages into its numerator and denominator separately. One could deform contour of inverse Hankel transform to obtain "branch" cuts instead of poles for the normal mode contributions. However, the same result follows from normal mode theory with averaging.

NORMAL MODE REPRESENTATION

Ahluwalia and Keller (1977) obtain the range independent normal mode representation of solution

$$u = \sum_{m=1}^{\infty} A_m \psi_m(r)\phi_m(z). \tag{8}$$

It has terms satisfying the right radiation condition if the radial function is $\psi_m(r) = H_0^{(1)}(ka_m r)$. The coefficients depend on source depth according to $A_m = \phi_m(z_0)$ where vertical equation is

$$[\partial_z^2 + k^2(n^2 + a_m{}^2)]\phi_m(z) = 0. \tag{9}$$

The asymptotic behavior of vertical eigenvalues is

$$a_m{}^2 \sim -h^{-2}(2\pi m + \theta)^2, \quad m \to \infty. \tag{10}$$

Set $q = k^2n^2$. Clearly the eigenvalues $\alpha_m = a_m^2$ will change as q changes. One can quantify this as a functional derivative. First, rewrite the z equation (9) (' is z differentiation)

$$\phi_q'' + (q - \alpha_q)\phi_q = 0 \tag{11}$$

Let q vary according to $q \Rightarrow q + \varepsilon p$, then one can compute the Fréchet derivative (see Ito *et al.* (1987))

$$\phi_{q,p} = d\phi(q,p) = d\phi(q)[p] = \left.\frac{\partial\phi_{q+\varepsilon p}}{\partial\varepsilon}\right|_{\varepsilon=0}. \tag{12}$$

Differentiate entire equation (11) with respect to ε to obtain

$$\phi''_{q,p} + (q - \alpha_q)\phi_{q,p} + (p - \alpha_{q,p})\phi_q = 0. \tag{13}$$

The inner product is $(f,g) = \int_{-h}^{0} f(z)g(z)dz$. For normalized eigenvalues, one differentiates the normality condition to find that

$$0 = (\phi_q, \phi_q)_{,p} = 2(\phi_{q,p}, \phi_q) . \tag{14}$$

Take inner product of ϕ_q with differentiated equation (13), and integrate by parts with respect to z. Since the boundary conditions for ϕ_q are constant, they carry over to $\phi_{q,p}$. This gives Fréchet derivative of eigenvalues

$$d\alpha(q)[p] = \alpha_{q,p} = (p\phi_q, \phi_q). \tag{15}$$

As a simple demonstration, take sound speed constant $q = 0$, $\theta(0) = \theta(\pi) = 0$, $k = 1$, and $h = \pi$, then $\alpha = \alpha_m = -1$ and $\theta = mz$. (15) becomes $d\alpha_m(q)[p] = 2/\pi\int_0^\pi p(z)\sin^2 mz\, dz$.

Also, one may apply (15) to compute average contribution from each mode, if the probability distribution of the speeds can be reparametrized by $\alpha = \alpha_m$. If $q = q(\omega)$, $p = \partial q / \partial\omega$, then one may compute the α dependence by $d\alpha_m(q)[p]dP(\alpha_m) = dP(\omega)$.

Turn now to the calculation of the average mode's contribution when the sound speed is constant, but statistically uncertain. This contribution will now depend on many horizontal wave numbers rather than just one. If $b_n = (n\pi + \theta)/h$ and $a_n = \sqrt{k^2 - (n\pi + \theta)^2 h^{-2}}$, the series (8) becomes

$$u(r,z) = \sum_{n=0}^{\infty} \sin b_n z_0 \, \sin b_n z \; H_0^{(1)}(a_n r), \tag{16}$$

For a rigid bottom condition pick $\theta = 1/2$. Take a uniform probability distribution over a small interval: $dP(k) = 1/(2\varepsilon)$, $k_0 - \varepsilon < k < k_0 + \varepsilon$. Three cases arise according to whether the eigenvalue for the probabilistically distributed mode is always positive, always negative, or spread around zero. The last case includes the previous two, so consider it. Suppose that the sound speed distribution is such that the eigenvalues are in a small band centered about zero: $k_0 = (n\pi + \theta)/h$, $\kappa = k - k_0$. One finds $a_n = [2k_0\kappa]^{1/2}$. Suppose r is large enough that one may use the asymptotic form

$$H_0^{(1)}(a_n r) \approx \frac{1-i}{\kappa^{1/4}}\sqrt{\frac{1}{\pi r\sqrt{2k_0}}}\exp i\left(\sqrt{2k_0}\,\kappa^{1/2} r\right) \tag{17}$$

For a single term of (16), only the Hankel function is affected by variation in eigenvalues. Thus the average of a term of (16) can be written as a product of the sines times an integral involving the Hankel function. Let $s = r\sqrt{2k_0\kappa}$, that integral becomes

$$\int H_0^{(1)}(a_n r)dP(k) \approx \frac{1-i}{\varepsilon r^2 2k_0\sqrt{\pi}}\left\{\int_0^{r\sqrt{2k_0\varepsilon}} e^{is} s^{1/2}\, ds - i\int_0^{r\sqrt{2k_0\varepsilon}} e^{-s} s^{1/2}\, ds\right\} =$$

$$= \frac{i+1}{\varepsilon r^2 2k_0\sqrt{\pi}}\left\{\tfrac{i+1}{\sqrt{2}}\,\gamma(3/2, -ir\sqrt{2k_0\varepsilon}) - \gamma(3/2, r\sqrt{2k_0\varepsilon})\right\} \tag{18}$$

These incomplete gamma functions appear instead of Hankel functions for ψ_n in (16).

Uncertainty in the bottom condition in the equations (16) give another example. One may model the uncertainty in the difference between the velocity of two fluid layers by random bottom boundary condition. Set $k = (n\pi + \theta_0)/h$, $dP(\theta) = d\theta/2\varepsilon$, $\theta_0 - \varepsilon < \theta < \theta_0 + \varepsilon$, $ht = \theta - \theta_0$. Thus $b_n = k + t$,